# Exam Quantum Physics 2

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| Date | 5 July 2016 |
| Room | A. Jacobshal 01 |
| Time | 14:00 - 17:00 |
| Lecturer | D. Boer |

- Write your name and student number on every separate sheet of paper
- Raise your hand for more paper
- You are allowed to use the book "Introduction to Quantum Mechanics" by Griffiths
- You are *not* allowed to use print-outs, notes or other books
- The weights of the four exercises are given below
- Illegible handwriting will be graded as incorrect
- Good luck!

**Weighting**

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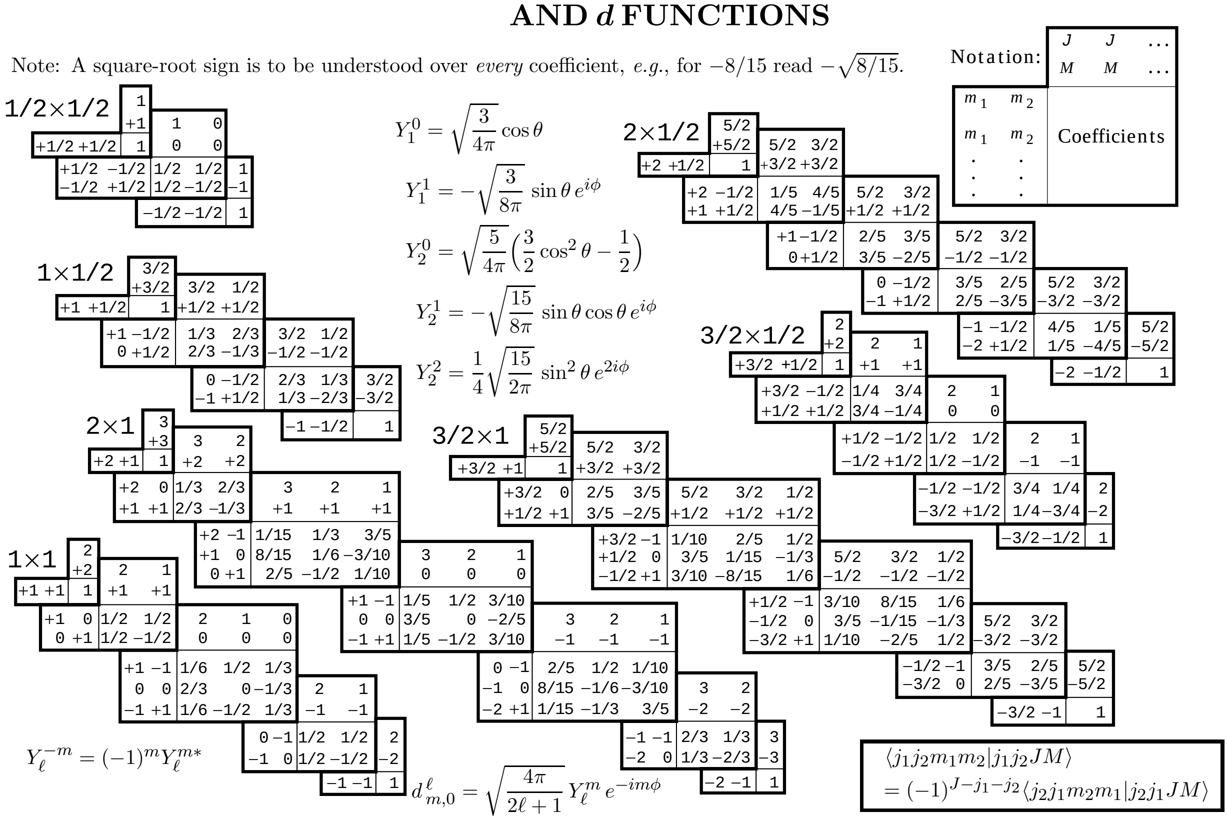
|---|---|---|---|---|---|---|---|
| 1a) | 6 | 2a) | 6 | 3) | 15 | 4a) | 7 |
| 1b) | 10 | 2b) | 6 | | | 4b) | 8 |
| 1c) | 10 | 2c) | 12 | | | | |
| 1d) | 10 | | | | | | |

$$\text{Result} = \frac{\sum \text{points}}{10} + 1$$

## Exercise 1

(a) Consider a system which involves two coupled angular momenta $\vec{L}_1$ and $\vec{L}_2$, in a state with quantum numbers $l_1 = 2$ and $l_2 = 1$. After addition of angular momentum one considers states of the total angular momentum $\vec{L} = \vec{L}_1 + \vec{L}_2$ with quantum numbers $l$ and $m$. Use the table below to write down the Clebsch-Gordan decomposition of the state $|l_1, l_2; l, m\rangle = |2, 1; 1, 0\rangle$ of total angular momentum $l = 1$ in terms of $|l_1, l_2; m_1, m_2\rangle$.

### 34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND *d* FUNCTIONS




(b) Demonstrate that $\left[\vec{L}^2, z\right] = 2i\hbar\,(xL_y - yL_x - i\hbar z)$.

(c) In case of an atom in a constant, uniform external electric field along the $\hat{z}$ direction, one has to calculate the matrix elements $\langle nlm|z|n'l'm'\rangle$. Demonstrate that these matrix elements vanish for $m' \neq m$, and explain that the behavior of the states $|n, l, m\rangle$ under parity transformations demands that for even $l + l'$ these matrix elements vanish.

(d) Explain why $j$ is not a good quantum number in case of a constant, uniform external magnetic field, and discuss (without calculations) what is the consequence of such a field for the time dependence of a state with given $j$ at $t = 0$?

**Exercise 2**

Consider a two-dimensional square well potential:

$$V(x,y) = \begin{cases} 0 & \text{for } 0 \le x \le a \text{ and } 0 \le y \le a \\ \infty & \text{elsewhere} \end{cases}$$

(a) Give the explicit expressions for the stationary states and the corresponding energies. Indicate the condition for degeneracy.

Next introduce the perturbation:

$$H'(x,y) = \begin{cases} -\alpha\delta(x - a/6) & \text{for } 0 \le x \le a \text{ and } 0 \le y \le a \text{ and } \alpha > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(b) When using degenerate perturbation theory for the degenerate excited states one generally has to consider off-diagonal matrix elements of $H'$. Use a symmetry argument to specify a basis on which the specific perturbation $H'$ given above has to be diagonal.

(c) Calculate in first order perturbation theory the correction(s) to the unperturbed energy of the first excited energy level. Recall that $\sin(\pi/3) = \sqrt{3}/2$ and $\sin(\pi/6) = 1/2$. You may make use of the following integral (valid for any integer $n$):

$$\int_0^a \sin^2\left(n\frac{\pi x}{a}\right) dx = \frac{a}{2}.$$

Indicate for which $\alpha$ values the result is expected to be valid.

**Exercise 3**

Consider the delta-function potential

$$V(x) = -\alpha\delta(x),$$

where $\alpha$ is a positive constant. This potential admits one bound state solution with energy $E = -m\alpha^2/(2\hbar^2)$. Determine the best approximation to the energy of this bound state that one can achieve with the variational method using the trial wave function

$$\psi_T(x) = \begin{cases} A\,(b - x) & \text{for } 0 \le x \le b \\ A\,(b + x) & \text{for } -b \le x \le 0 \\ 0 & \text{elsewhere} \end{cases}$$

with normalization $A = \sqrt{3/2}\,b^{-3/2}$ and positive parameter $b$. Explain whether the answer is as one expects for the variational method.

**Exercise 4**

Consider the Hamiltonian $H = H_0 + H'(t)$, where $H'$ is a time-dependent perturbation that is nonzero for $t \geq 0$. Let $\psi_n^{(0)}$ be the orthonormal set of eigenstates of $H_0$ with energies $E_n^{(0)}$, i.e. $H_0\,\psi_n^{(0)} = E_n^{(0)}\,\psi_n^{(0)}$.

(a) Show that with the following expansion on the states $\psi_n^{(0)}$

$$\psi(t) = \sum_n c_n(t)\,\psi_n^{(0)}\,e^{-i\,E_n^{(0)}t/\hbar},$$

the coefficients satisfy

$$\dot{c}_m(t) = \frac{1}{i\hbar}\sum_n c_n(t)\,e^{i\,(E_m^{(0)}-E_n^{(0)})t/\hbar}\,H'_{mn},$$

where $H'_{mn} = \langle\psi_m^{(0)}|H'|\psi_n^{(0)}\rangle$.

(b) Consider the case where $H'(t) = V(r)\theta(t)$ for a two-level system consisting of states $\psi_1$ and $\psi_2$, such that $\langle\psi_i|V(r)|\psi_j\rangle \neq 0$ only for $i \neq j$. Derive, to first nontrivial order in time-dependent perturbation theory, what the probability is to be in state $\psi_2$ as a function of time if the system is in state $\psi_1$ for $t < 0$.